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Report 1

Make digital filters run faster

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TABLE OF CONTENTS

1	Overview	3
1.1	Digital filters	3
1.2	Digital Transforms	3
2	The goal of this paper	4
3	Faster Filter Algorithms	4
3.1	Decimation	4
3.2	FIR Filter Structure	6
3.2.1	Direct-form FIR structure (tapped delay line)	7
3.2.2	Symmetric direct-form FIR structure.	7
3.2.3	Transposed direct-form FIR structure	8
3.3	IIR Filter Structure	9
3.3.1	Direct-form IIR structure	9
4	Faster microcontroller	9
4.1	Criteria related to microcontroller architecture	9
4.2	Criteria related to the speed of calculation	11
4.3	Economic criteria	14
4.4	Digital Signal Processor.	14
5	Faster type of computation	16
5.1	Fixed point vs floating point	16
6	Programming techniques	17
6.1	Look Up Table (LUT)	17
6.2	Programming Language	18
6.3	Avoiding slow functions	18
	Table of Figures	20
	Table of Equations	20

Note: This report is based on a Template by Prof. dr. ing. Miha Ioan. Contents with background colour were added by Stefan Feilmeier.

1 OVERVIEW

In this document, we will be focused on the most important algorithms used in DSP:

1.1 DIGITAL FILTERS

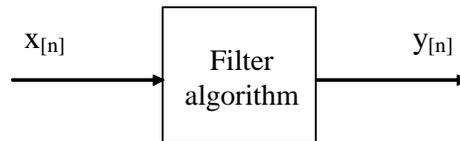


Fig. 1 Digital filter

It is well known what the digital filter is: a linear algorithm. There are two types:

- **Non-recursive filters** also named Finite Impulse Response (FIR)

$$y[n] = \sum_{k=0}^{N-1} h[k] \times x[n-k]$$

Eq. 1 Non-recursive filters

FIR filter requires N multiplications and $N - 1$ additions at any input sampling moment (for each new sample in input data stream).

- **Recursive filters** also named Infinite Impulse Response (IIR)

$$y[n] = \sum_{k=0}^{N-1} h[k] \times x[n-k] - \sum_{i=1}^M b[i] \times y[n-i]$$

Eq. 2 Recursive filters

IIR filter requires $N + M$ multiplications and $N + M - 1$ additions for each new sample in input data stream.

1.2 DIGITAL TRANSFORMS

- **Discrete Fourier Transform (DFT)**

$$X_{TFD}[n] = \sum_{k=0}^{N-1} x[k] \times e^{-j\frac{2\pi nk}{N}}$$

Eq. 3 Discrete Fourier Transform

DFT requires N^2 multiplications and $2 \times (N - 1)$ additions.

- **Fast Fourier Transform (FFT)**

$$\left\{ \begin{array}{l} X_{TFD}[n] = \sum_{k=0}^{N-1} x[k] \times e^{-j\frac{2\pi nk}{N}} \\ \text{for } N = 2^P \end{array} \right.$$

Eq. 4 Fast Fourier Transform

FFT requires $N \times \log(N)$ multiplications and $2 \times (N - 1)$ additions.

- **Cosine Transform (CT)**

$$y[n] = \lambda(n) \sum_{k=0}^{N-1} x[k] \times \cos \frac{\pi n(2k + 1)}{2N}$$

Eq. 5 Cosine Transform

CT requires N multiplications and $N - 1$ additions.

2 THE GOAL OF THIS PAPER

Needs required by the last decade industry, have made almost any product to incorporate a computer system (embedded system) that controls a process. Usually microcontrollers are connected to fast processes from which they are acquiring signals, process them and then return the other signals for process control. This has forced researchers to seek solutions to provide real-time work requirement.

Real-time system must be viewed in the context of many tasks for the computing system. Knowing controlled process, the programmer will define the time limit for each task that must be fulfilled. With this in mind, it is obvious that the calculations need to be increasingly faster. Therefore we look in this paper at some ways to make possible to use microcontrollers in real-time systems, working in high-speed conditions.

We group the solutions into four clusters: faster filter algorithms, faster filter structure, faster microcontroller, and faster programming techniques.

3 FASTER FILTER ALGORITHMS

3.1 DECIMATION

History: The term “decimation” comes from ancient romans. If at that time, an army lost the battle, the emperor punishes them: put all the soldiers in line, and every 10th soldier was killed. In Latin language “decimus” means “the tenth”. Now, we do not have soldiers, but we have stream of samples (numbers) coming from Analog to Digital Converter.

In this context, we shall find reason to eliminate some sample from the stream. Not always the tenth, but depending on context, can be removed the second, the third, or more complicated rules can be used to eliminate samples. Nevertheless, today the term “decimation” is used in all these cases of removing samples.

If a Low Pass Filter (LPF) is used in the path of a digital signal, it will decrease the width of the spectrum frequency band. The new spectrum will not cover the entire Nyquist interval $[0 \div Fs/2]$.

Consequently, it means we have no need for such a high sampling frequency of $Fs/2$, for a signal having a spectrum where the upper limit is less than $Fs/2$. For this reason, we can reduce the sampling frequency.

Example:

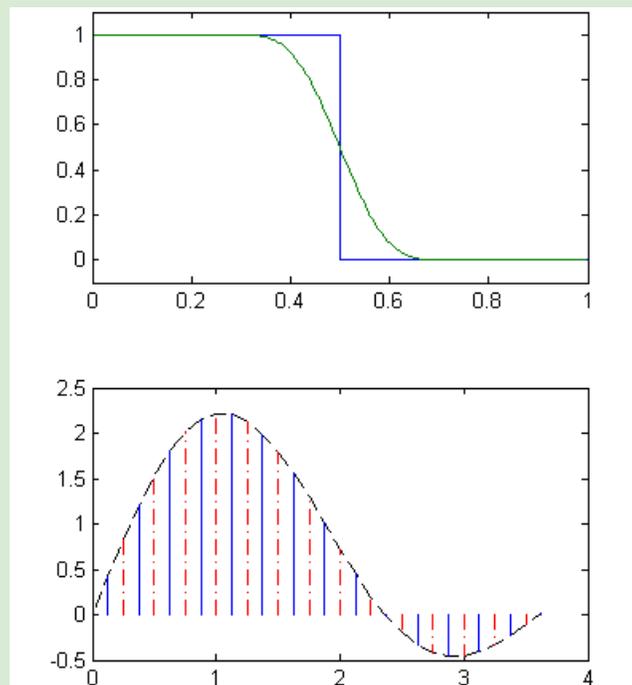


Fig. 2 Example of decimation

Where is the benefit in terms of computation? In fact the algorithm (the computation) is launched at every sampling moment, when a new input sample is delivered to filter. Therefore, we can define a new Number of multiplication/sample.

If the factor of decimation is not an entire one, the decimation problem becomes a little bit more complicated. For instance it cannot be eliminated “the 2.5th sample”, because such a sample does not exist. In these cases, we need to create them first. “The 2.5th sample” can be created by using interpolation techniques. The new samples can be seen in the example below, created mathematically by interpolation. It looks as the sampling frequency is 5 times bigger than the original Fs .

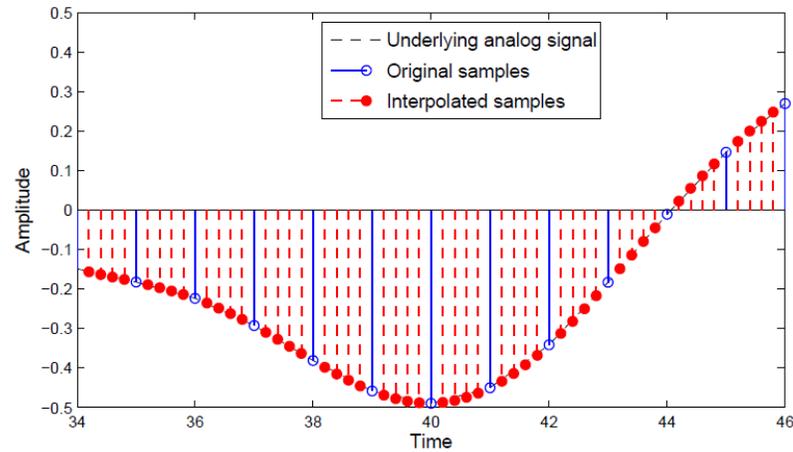


Fig. 3 Interpolation

Now another problem occurs in term of time computation. We have benefit using the decimation, but we lost time for interpolation. For this reason it is recommended to use LPF with pass band being $1/L$ of the Nyquist interval. These filters are called Nyquist filters. The use of such filters allows to very easy decimating: we shall take the L^{th} samples in the stream.

3.2 FIR FILTER STRUCTURE

FIR (Finite Impulse Response) filters typically require higher orders than IIR filters to achieve proper results. Even if FIR filters seem to need less processing power (N multiplications and $N - 1$ additions) than IIR filters ($N + M$ multiplications and $N + M - 1$ additions), a much higher order can result in the demand of more processing power and memory consumption in total than equivalent IIR filters would need.¹

FIR filters are still used, because they are very easy and straight-forward to implement and can easily be designed to have a linear phase response, resulting in no phase shift across the frequency band, which can be required for certain applications (like the display of an oscilloscope, video processing and so on).²

¹ WikiBooks Signal Processing/Digital Filters, http://en.wikibooks.org/wiki/Signal_Processing/Digital_Filters

² The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith, Ph.D., Chapter 19: Recursive Filters, Phase Response, <http://www.dspguide.com/ch19/4.htm>

3.2.1 Direct-form FIR structure (tapped delay line)

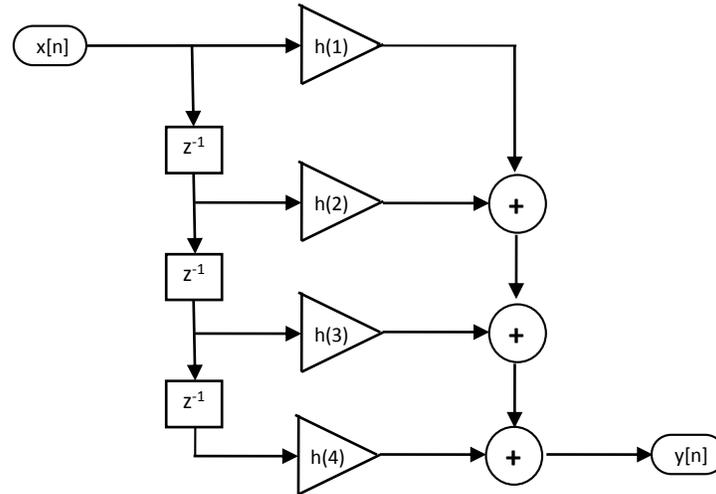


Fig. 4 FIR filter (order 3) implemented in direct-form

$$y[n] = (x[n] \times h_{(1)}) + (x_{[n-1]} \times h_{(2)}) + (x_{[n-2]} \times h_{(3)}) + (x_{[n-3]} \times h_{(4)})$$

Eq. 6 FIR filter (order 3) implemented in direct-form

- Matlab command: `dfilt.dffir`
- Advantages:
 - Easy, straight forward implementation
 - Fast implementation and application on simple filter problems
- Disadvantages:
 - For symmetric filters many calculations are computed redundantly
 - Many additions at the end may cause problems when implementing the algorithm in hardware. (requires extra pipeline registers to reduce delay of adder tree and to achieve high throughput)³

3.2.2 Symmetric direct-form FIR structure.

It is exploiting the symmetric coefficient equality.

It is assumed, that the coefficients are symmetric: $h_{(1)} = h_{(4)}$; $h_{(2)} = h_{(3)}$

³ <http://www.edaboard.com/thread30463.html>

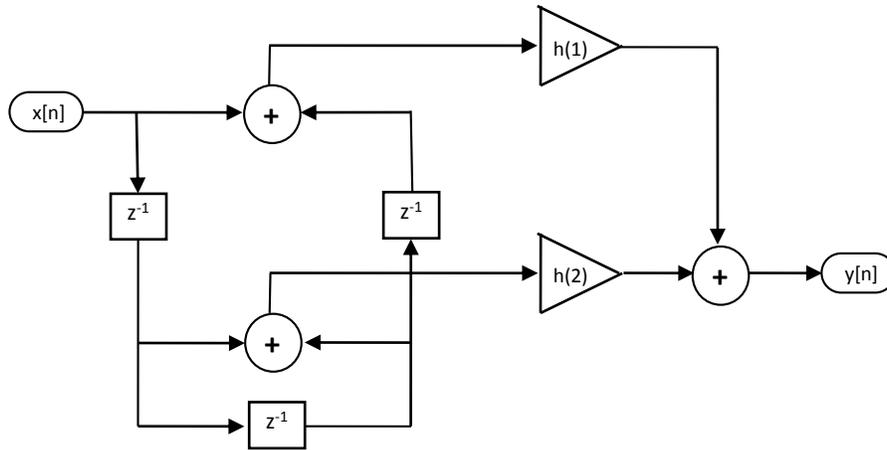


Fig. 5 FIR filter (order 3) implemented in symmetric direct-form

$$y[n] = [(x[n] \times h_{(1)}) + (x_{[n-3]} \times h_{(1)})] + [(x_{[n-1]} \times h_{(2)}) + (x_{[n-2]} \times h_{(2)})]$$

Eq. 7 FIR filter (order 3) implemented in symmetric direct-form

- Matlab command: `dfilt.dfsymfir`
- Advantage:
 - Less computing power required
- Disadvantage:
 - Increased algorithm complexity

3.2.3 Transposed direct-form FIR structure

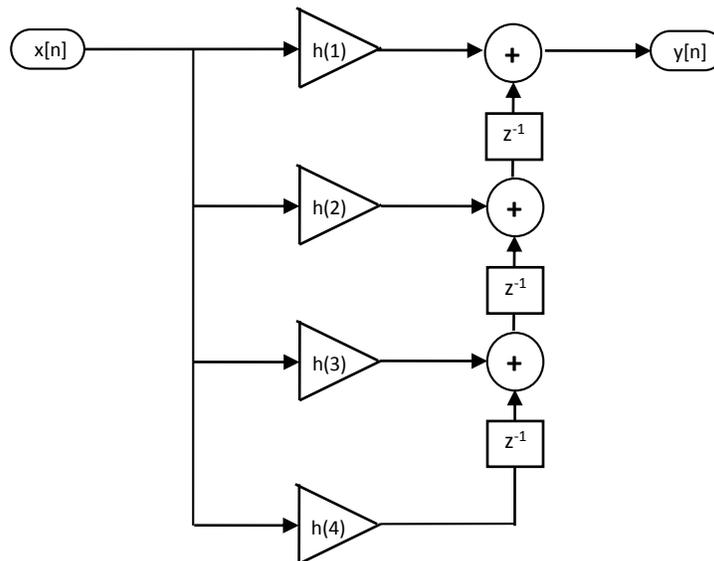


Fig. 6 FIR filter (order 3) implemented in transposed direct-form

$$y[n] = \{[(x_{[n-3]} \times h_{(4)}) + (x_{[n-2]} \times h_{(3)})] + [(x_{[n-1]} \times h_{(2)})] + (x_{[n]} \times h_{(1)})\}$$

Eq. 8 FIR filter (order 3) implemented in transposed direct-form

- Matlab command: `dfilt.dffirt`
- Advantage:

- Easier and faster implementation in hardware (many small additions in contrast to one big addition at the end)

3.3 IIR FILTER STRUCTURE

IIR (Infinite Impulse Response) can provide higher quality filters, requiring less computations and less memory consumption compared to FIR filters. But because of the internal feedback mechanism, linear response phase is not easily achieved and the feedback can also cause problems in finite-precision arithmetic, as inaccuracies are carried forward.

3.3.1 Direct-form IIR structure

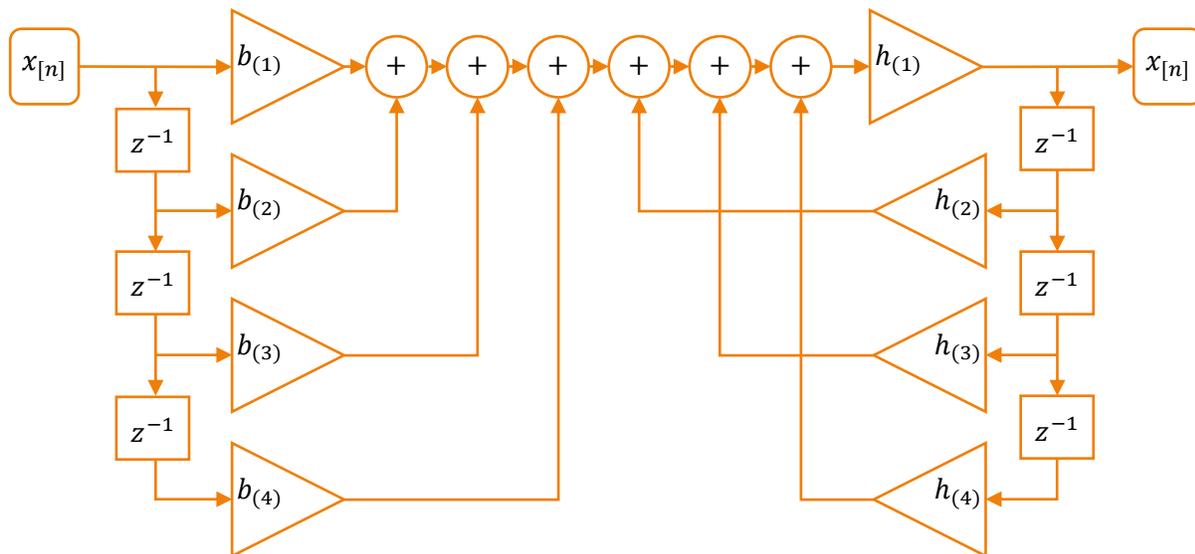


Fig. 7 IIR filter (order 3) implemented in direct-form

- Matlab command: `dfilt.df1`
- Advantages:
 - Easy, straight forward implementation
 - Fast implementation and application on simple filter problems

4 FASTER MICROCONTROLLER

4.1 CRITERIA RELATED TO MICROCONTROLLER ARCHITECTURE

The problem can be formulated as follows: what type of microcontroller is best suited to my application? The objective criteria are related to the technical requirements of the application. Here are some of them: the number of pins, the capsule type, voltage, volume flash memory, the amount of RAM, specific ports: I2C, UART, ADC, etc.

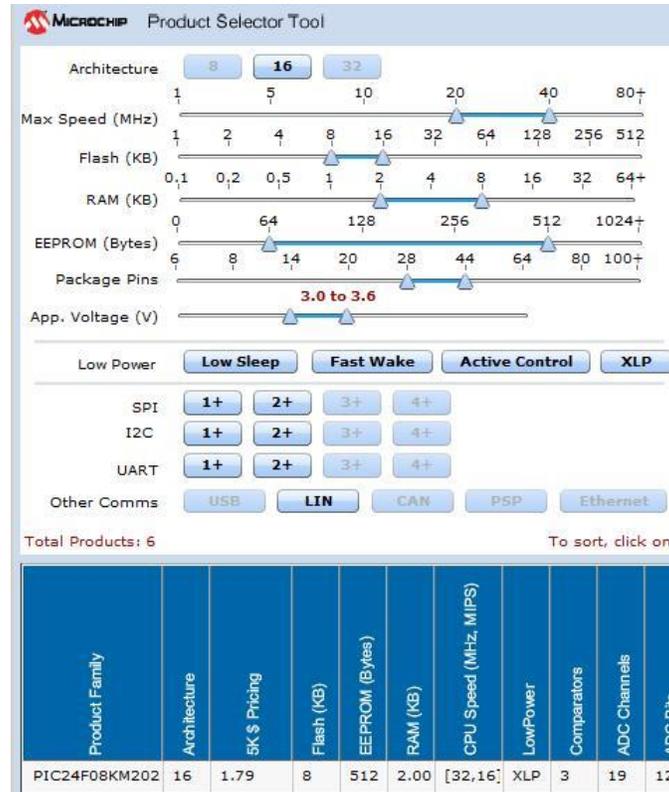


Fig. 8 Software tool for choosing appropriate μC

Searching an optimal microcontroller is not a very easy, especially if you are just starting out in their use. The company Microchip comes to the rescue with a very useful software tool for choosing the optimal microcontroller. It is accessible on the website of the company⁴. A part of this interface is shown in Fig. 8. It is noted that we can select the search area, using “graphic potentiometers” (slider). We can always narrow the search area until they reach the desired microcontroller technically.

Here is a summary of the main features of Microchip μC families:

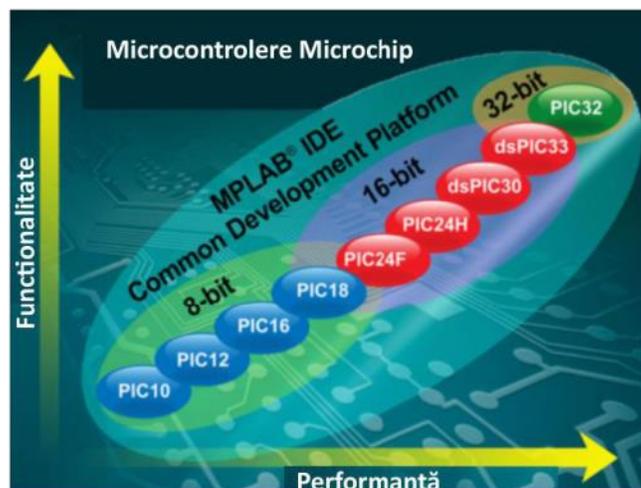


Fig. 9 Overview of Microchip μC families

⁴ www.microchip.com/productselectiontool

Data bus	Instruction length	Family/ Type	Performance
8 bit	12 bit	PIC12/ PIC12F675	0,064k RAM, 1 kB Flash; 20 MHz; 6 I/O pin; 2 Timers; 7 Interrupt sources; 10 bit ADC/4 ch; 32 Base Instructions
	14 bit	PIC16/ PIC16F877	0,368k RAM, 8 kB Flash; 20 MHz; 14 Interrupt sources; 3 Timers; 2 PWM channels; 1 USART; 10 bit ADC/8 ch; 35 Base Instructions,
	16 bit	PIC18/ PIC18F452	1,6k RAM, 32 kB Flash; 40 MHz; 2 PWM channels; 18 Interrupt sources; 4 Timers; 1 USART; 10 bit ADC/8 ch; 75 Base Instructions, 8x8 hardware multiplier;
16 bit	24 bit	PIC24/ PIC24FJ128- GA006	16k RAM, 256 kB Flash; 40 MIPS; 35 I/O pin; 2 UART; 2 SPI; 2 I2C; 10 bit ADC/13 ch; 5 PWM channels; 43 Interrupt sources; 5 Timers; 76 Base Instructions, 17x17 hardware multiplier;
		dsPIC30/ dsPIC30F4012	16k RAM, 256 kB Flash 30 MIPS; 35 I/O pin; 2 UART; 2 SPI; 2 I2C; 10 bit ADC/13 ch; CAN bus; 45 Interrupt sources; 5 Timers; 8 PWM channels; 76 Base Instructions, 17x17 single cycle fractional/ integer multiplier;
		dsPIC33/ dsPIC33FJ256- GP506	30k RAM, 256 kB Flash; 40 MIPS; 85 I/O pin; 2 UART; 2 SPI; 2 I2C; 10 bit 2 ADC/32 ch; CAN bus; 117 Interrupt sources; 5 Timers; 8 PWM channels; 83 Base Instructions, 16x16 single cycle fractional/ integer multiplier;
32 bit	128 bit	PIC32/ PIC32MX360- F512L	128k RAM; 512 kB Flash; 80 MHz / 1,65MIPS/MHz O Instructions/Cycle; Cache memory; 16ch DMA; Ethernet; 2CAN buss; Full speed USB; Multitasking embedded control; Free MPLAB XC Compiler

Fig. 10 Summary of PIC families performances

4.2 CRITERIA RELATED TO THE SPEED OF CALCULATION

Most exciting applications of microcontrollers are called “real time”. In those applications the reaction/response of the system must be fast enough to be useful to the application requirements. For example if the microcontroller system equips the steering system of a cruise missile with a speed of 400 m/s, it is vital that once the microcontroller receives information about the current position and direction of an obstacle, it takes and passes decision to change direction in the shortest time possible in real time. Decision shall be taken after following an algorithm – so after some calculations. It is vital that these calculations are made quickly. If we have a speech recognition system, the samples are read with a sampling frequency of 8 kHz, to the algorithm is required to make all the calculations in the interval between two samples ($T_e = 1/8000s = 0,125ms$).

Typically, the algorithms are linear, calculations involved are additions and multiplications. Knowing the algorithm, we know how many multiplications and how many additions have to be made. It is natural therefore to have an overview of the computing time assigned for an addition and for a multiplication by each family of microcontrollers. This is summarized in Fig. 11.

Data bus	Instruction length	Family/ No of instructions	Calculation time (cycles per instruction)					
			Addition			Multiplication		
			8 bit + 8 bit	16 bit + 16 bit	float + float	8 bit x 8 bit	16 bit x 16 bit	float x float
8 bit	12 bit	PIC12 32 Instructions	1	3 soft	130 soft	2 soft	4 soft	526 soft
	14 bit	PIC16 35 Instructions	1	3 soft	130 Soft	2 soft	4 soft	526 soft
	16 bit	PIC18 75 Instructions	1	3 soft	115 soft	1	4 soft	451 soft
16 bit	24 bit	PIC24 76 Instructions	1	1	88 soft	1	1	90 soft
		dsPIC30 76 Instructions	1	1	84 soft	1	1	90 soft
		dsPIC33 83 Instructions	1	1	84 soft	1	1	90 soft
32 bit	128 bit	PIC32 124 Instructions Multitasking	1	1	64 soft	1	1	49 soft

Fig. 11 Calculation speed of microcontrollers

For the beginning we analyze the computational time for additions. All families of microcontrollers have the instruction set on the assembly: ADDWF = addition of two variables, or ADDWFC = addition of a variable with a constant. This means that if the numbers are on one byte and the data bus is 8 bit hardware, the addition will take only one instruction cycle. Instead if we have to add numbers on two byte and data bus is 8 bits, then it will require more instruction cycles. If the data bus is 16 bits then the addition of two 16 bit numbers will be done in a single instruction.

Let us continue the analysis of data in Fig. 11 with the multiplication. PIC18 family is the first to have incorporated an 8x8 bit hardware multiplier. This means that it performs a multiplication in one single cycle instruction (there are MULs instruction in the instruction set). PIC24 and dsPIC30 families, have a 17x17 hardware multiplier that allows multiplication of two 16-bit numbers in a single instruction cycle.

```

8      unsigned int  d, e, f; // 2 octeti
9      float        x, y, z; // virgula flotanta
10
11      d = 12;      // 3 cicli instructiune
12      e = 2333;   // 3 cicli instructiune
13      f = d + 1;  // 3 cicli instructiune
14      f = d + e;  // 3 cicli instructiune
15      f = d + 11; // 3 cicli instructiune
16      f = d * e;  // 4 cicli instructiune
17
18      x = 3.66666; // 6 cicli instructiune
19      y = 2.33333; // 6 cicli instructiune
20      z = 2.3 + 1.1; // 6 cicli instructiune
21      z = x + 1.1; // 141 cicli instructiune
22      z = x * y;   // 130 cicli instructiune
23      z = x * 1.2; // 536 cicli instructiune
24      z = x * y;   // 526 cicli instructiune
    
```

Fig. 12 Code sequence used for Fig. 11

The data in Fig. 11 was collected by running portion of code in Fig. 12 and then using MPLAB SIM simulator. After setting the Debug menu simulator for analysis we use the disassembly listing window (Fig. 14), or Program Memory window (Fig. 13). We find the number of instructions using the Step Into

button for stepping forward in the program. In Fig. 14 it is observed that the PIC24 multiplies the 2 bytes numbers in a single cycle.

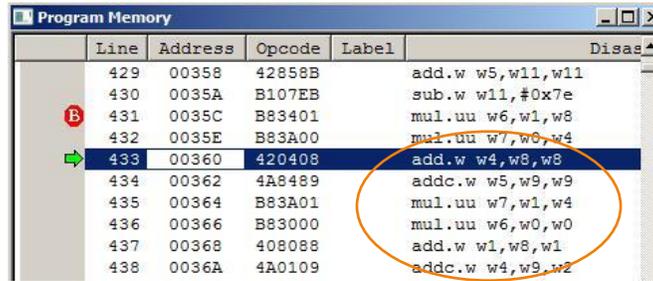


Fig. 13 Program Memory Window

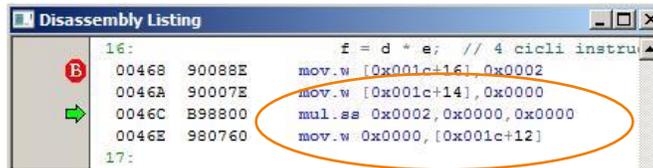


Fig. 14 Disassembly Listing Window

Of the two windows we can find the number of instruction routines. It is not fair to judge the speed of calculation in “numbers of instructions” per multiplication routine because it includes loops that are dependent on the numbers that are multiplied. The best way to judge the speed of calculation is the analysis of computation time for resulting executable code with MPLAB SIM simulator Simulator Trace window, as shown in Figure 2.15.

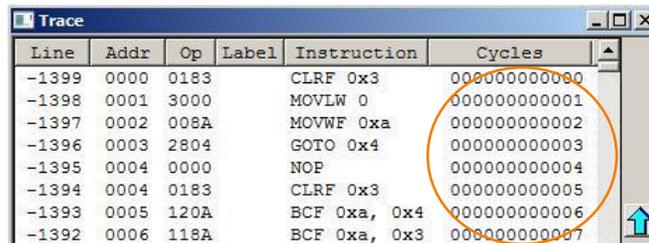


Fig. 15 Simulator Trace Window

As shown in Fig. 11, the PIC 16 PIC12 families do not have a multiplication instruction, so it can perform multiplications only by software routines; the multiplication is done by repeated additions. For example, a PIC16 microcontroller performs multiplication of two numbers represented by 2 bytes in 4 instruction cycles. In Fig. 11 cells the words “soft” means that the operation is not carried out in a single instruction, but is executed in a routine.

A bigger problem is the addition and multiplication of floating point numbers. Although they provide high accuracy, time to make an addition or a multiplication is very high. All operations with variables of type float are made with very long soft routines, for two reasons:

- On completion of this document, the Microchip microcontroller’s family does not have embedded FPU (Floating Point Unit), so operations with floating-point numbers cannot be achieved in hardware, but only through software emulation.

- The numbers in floating point format in IEEE754 standard must be “unwrapped first” to be brought to a representation in the form of integers, the only ones that ALU knows to perform calculations with. After calculations, they must be “wrapped” again in floating point format.

The number of instruction cycles may vary as specified in the table, because it is dependent on the values of the operands (especially when they are of particular values such as 0, 1, NaN, etc.), as well as the type of compiler. Another measure of compiler performance to handle floating-point numbers is the number of instruction cycles to calculate the value of math functions such as $\sin(x)$, $\tan(x)$, $\ln(x)$, etc., even these algorithms are used in real-time.

Finally, returning to the big picture provided by Fig. 11, we see that there are operations conducted on various types of data: 8 bit, 16 bit or float that can be performed in a single instruction cycle or in many. It becomes clear that the programmer must be responsible to choose the right type of variables, in order for the computing speed to be as high as possible. So if he wants to work with 8-bit variables, then he may choose a microcontroller with an 8-bit bus, and the program would declare variables of type char or unsigned char. If it is forced to work with greater accuracy and should use 16-bit variables, then it is recommended to choose a microcontroller with 16-bit data bus (PIC24) and in the program to declare variables of type int. If he wants high accuracy, it is natural to think about using float variables. As seen in the table, they can only be used when there are large reserves of computation time constraints.

The need to perform quick calculations is actually the “engine” that pushed the producer’s microcontroller families to realize architectures with increasingly high performance, as we discuss the Digital Signal Processors chapter.

4.3 ECONOMIC CRITERIA

Along with technical criteria, there are others, at least as important: microcontroller cost, compiler cost, user experience, etc. These criteria are important both for the student who performs teaching or hobby projects in its budget and for industrial projects conducted by specialized companies in producing embedded systems with microcontrollers.

In conclusion, choosing the right microcontroller is an important step and not very easy to do when you have to achieve competitive applications, high complexity and high performance.

4.4 DIGITAL SIGNAL PROCESSOR.

General purpose processors based on the “Von Neumann architecture”⁵ are optimized for data manipulation, like it is needed in word processing, database management, and other applications alike. For these usage scenarios, they need to be optimized for data movement and value testing operations, rather than mathematical calculations. While mathematical calculations are also happening in those applications, they are infrequent.

In comparison, digital signal processing algorithms like digital filters and Fourier analysis, are almost entirely based on mathematical calculations like multiplications and additions. Therefor a special kind of

⁵ Described by John von Neumann in 1945.

microprocessor, optimized for mathematical calculations, had to be designed: the DSP (Digital Signal Processor).⁶

Further optimizations of DSPs in comparison to general purpose processors are required, as DSPs are generally used to process continuous signals:

- Execution of an algorithm within a predictable execution time (“real-time processing”)
- Implementation of a circular buffer to ease handling of the signal’s stream of numbers
- Extra wide accumulator register to reduce round-off errors with multiple fixed-point operations
- Shadow registers for fast context switching to handle interrupts quickly
- Optimized performance of the algorithm’s loop:

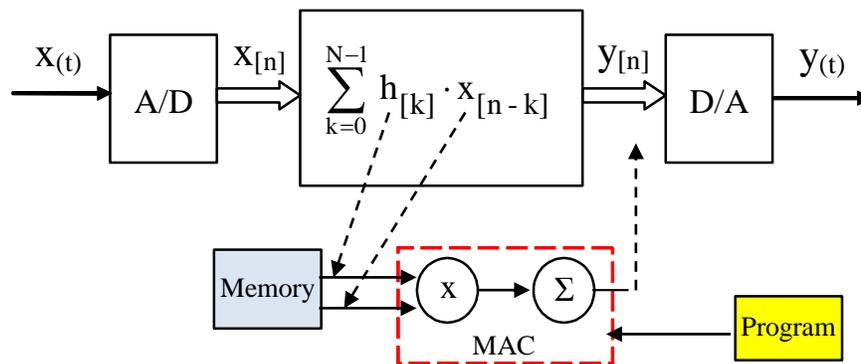


Fig. 16 DSP device performing the filter algorithm

The two operands, $h_{[k]}$ and $x_{[n-k]}$, are directed to a multiply operation, and the result is summed. This process is built into the chip in that two separate data memory accesses are allowed to feed a single-cycle MAC (Multiplier-Accumulator). The entire process must occur under program control to direct the correct operands to the multiplier and to save the accumulated result, as needed. Since the memory and the MAC are independent, the DSP device can perform two memory moves, a multiply and an accumulation, and two address updates in a single operation. These activities take only one cycle!

As an example, an FIR filter executes the following steps:

1. Obtain sample from ADC + generate interrupt
2. Handle interrupt
3. Move sample into input signal’s circular buffer + update buffer pointer
4. Zero accumulator
5. Loop over each coefficient
6. Fetch coefficient from coefficient’s circular buffer + update buffer pointer
7. Fetch sample from input signal’s circular buffer + update buffer pointer
8. Multiply coefficient by sample
9. Add product to accumulator
10. Move accumulator value (output sample) to holding buffer
11. Move holding buffer to DAC

⁶ The Scientist and Engineer’s Guide to Digital Signal Processing, Steven W. Smith, Ph.D., Chapter 28: Digital Signal Processors, <http://www.dspguide.com/ch28.htm>

Steps 5 to 9 are repeated many times, once per each coefficient in the filter. DSPs like the “Harvard Architecture”⁷ or the “Super Harvard Architecture” (SHARC)⁸ are designed to perform those operations in parallel (compared to general purpose processors, which would execute them in serial), in some cases all operations within the loop can be completed in one single clock cycle. For example an efficient FIR filter program using 100 coefficients could be expected to execute in about 105 to 110 cycles per sample, which is 100 coefficient loops with one clock cycle each, plus some overhead for beginning and ending the loop.

5 FASTER TYPE OF COMPUTATION

5.1 FIXED POINT VS FLOATING POINT

Computers are working in binary, with 0s and 1s only. This makes it easy to store integers, so called “fixed point” numbers:

$$0_{10} = 0_2, 1_{10} = 1_2, 2_{10} = 10_2, 3_{10} = 11_2, 4_{10} = 100_2, 5_{10} = 101_2, 6_{10} = 111_2, \dots$$

The problem comes up, when trying to save numbers with a fractional component (“with decimal places”). According to “IEEE Standard for Floating-Point Arithmetic” (IEEE 754, latest version from August 2008) those “floating point” numbers are represented in computers in the following way: (datatype “float” on 32 bit)

$$\begin{array}{ccccccc}
 31 & 30 & & & 23 & 22 & & & 1 & 0 \\
 [0] & [0 & 0 & \dots & 0 & 0] & [0 & 0 & \dots & 0 & 0] \\
 S \text{ (sign)} & & E \text{ (exponent)} & & & & M \text{ (significant/mantissa)} & & & &
 \end{array}$$

The standard defines a “hidden bit” in front of the mantissa which is always set to 1, followed by the comma, to always require the mantissa to be normalized, which is important for unique representation of numbers, not losing digits to leading zeros, etc.

The actual value of the floating point number is then calculated using:

$$x_{10} = -(1)^S \times (1, M_2) \times 2^{(E-127)}$$

The floating point standard allows the handling of extremely big ($E = 254 \Rightarrow 127$) and extremely small numbers ($E = 1 \Rightarrow -126$). ($E = 255$ and $E = 0$ are considered being “not a number” (NaN).) But in contrast to integers, which are always exact in their respective interval (resolution = 1), floating point numbers have a variable resolution. This means, that the interval between two consecutive numbers is different for different numbers, reaching from as high as 10^{+31} for a big exponent to as small as 10^{-31} for small exponents.

Mathematical operations with floating point numbers, require some special techniques compared to integers, and generally require treating exponent and mantissa separately. If the resulting value falls between two consecutive numbers, rounding will be required, which introduces errors on every operation, and decreases the signal-to-noise ratio (SNR). Also, as shown in Fig. 11 Calculation speed of

⁷ Developed under the leadership of Howard Aiken at Harvard University in the 1940s.

⁸ Termed by the company Analog Devices to describe the internal operation of their DSPs, which are extending the Harvard Architecture.

microcontrollers, these calculations have to be executed in software, if using a general purpose processor without FPU (Floating-Point Unit), which is adding an enormous overhead. This overhead (and many problems that are arising with the usage of fixed-point arithmetic) can be avoided using a specialized Digital Signal Processor as introduced in chapter 4.4.

To make digital filters run faster on processors without Floating-Point Unit, it is recommended to use fixed-point arithmetic, which can be calculated by a simple ALU (Arithmetic Logic Unit), instead of floating-point. In Matlab this can be achieved using the following command:

```
Hf = fdesign.lowpass(0.4,0.5,0.5,80);
Hd = design(Hf,'equiripple');
Hd.Arithmetic = 'fixed';
```

Using this command, Matlab quantizes all filter into a fixed-point representation, which is per default set to a wordlength of 16 bits, but can be adjusted to meet the required noise ratio.⁹ From then on, all calculations in Matlab are fulfilled using fixed-point arithmetic, adjusting and increasing the bit-size of its internal variables automatically.

6 PROGRAMMING TECHNIQUES

6.1 LOOK UP TABLE (LUT)

“Many applications in digital communication, speech processing, seismic signal processing and several other areas require large order FIR filters.”¹⁰ The number of multiply-accumulate (MAC) operations required per filter output increases linearly with the filter order. One approach to reduce computation time and power consumption of the many multiplications that are needed, is to use “memory-based structures” like a Look Up Table (LUT). Such a table is used to store all possible product values for each coefficient.

If A is a fixed coefficient and X is an input word with length L , there are 2^L possible product results of $C = A \times X$. The Look Up Table is filled with all those values, using X_i as memory address of each product result $A \times X_i$, so that X_i can be used directly as index to the table.

Example LUT for coefficient $A = 15$ (fixed-point arithmetic) and input wordlength $L = 2$ bit:

Address	X	Result
0_2	0_2	0_{10}
1_2	1_2	15_{10}
2_2	2_2	30_{10}
3_2	3_2	45_{10}

⁹ Digital Filters with MATLAB, Ricardo A. Losada, The MathWorks Inc. 2008, Chapter 7.2 “Fixed-point implementation” http://www.mathworks.com/tagteam/55876_digfilt.pdf

¹⁰ Implementation of Area Efficient Memory-Based FIR Digital Filter Using LUT-Multiplier, K.Purnima et al, (IJCSIT) International Journal of Computer Science and Information Technologies, Vol. 3 (2), 2012,3530-3535, <http://www.ijcsit.com/docs/Volume%203/Vol3Issue2/ijcsit2012030250.pdf>

6.2 PROGRAMMING LANGUAGE

Digital Signal Processing is nowadays not anymore only used in specialized DSP devices, but is used in all kinds of systems and system environments. As an example, most of today's mobile phones are running a Linux-kernel based Android operating system. An Android frontend developer usually uses Java for his applications, while the Linux-kernel is written mostly in C language with some parts in Assembly. Also other compiled and interpreted programming languages like C++, Python, etc. are available in this environment.

A programmer will for various reasons prefer to use the same programming language for each part of his application. The question is: does it make sense to write the complete DSP processing in Java? Or in C or Assembly?

There is no absolute answer to this question, as it highly depends on the specific system and economic constraints. As a rule-of-thumb it can be expected, that a subroutine that is written "in Assembly will be between 1.3 and 3.0 times faster than the comparable high-level program"¹¹ in C. So depending on the constraints it can or cannot be worth implementing some parts of the application in Assembly or, if they exist, consider using libraries that provide algorithms implemented in Assembly.

In general, not limited to DSP, it is always an appropriate idea to implement everything in the preferred and best fitting language for the environment and afterwards do a detailed profiling to find the bottlenecks. Those subroutines should then be considered to be optimized or rewritten in a lower-level language.

6.3 AVOIDING SLOW FUNCTIONS

As shown in 5.1 Fixed point vs floating point as well as in Fig. 11 Calculation speed of microcontrollers, floating-point operations require a lot of computing time compared to fixed-point (integer) arithmetic. Similarly, transcendental functions require huge calculation time when used in an algorithm. Therefore it is necessary to know about them and to try to avoid or replace them if possible.

Examples for transcendental functions are:

$$\sin(x), \cos(x), e^x, x^\pi, \log_c x \text{ for } c \neq 0,1$$

¹¹ The Scientist and Engineer's Guide to Digital Signal Processing, Steven W. Smith, Ph.D., Chapter 4: DSP Software, <http://www.dspguide.com/ch4/5.htm>

EQUATION 4-3

Maclaurin power series expansion for three transcendental functions. This is how computers calculate functions of this type, and why they execute so slowly.

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \dots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

Fig. 17 Maclaurin power series expansion for transcendental functions¹²

If the functions are needed for the application, techniques like Look Up Tables as discussed in 6.1 can be considered.

¹² See above, <http://www.dspguide.com/ch4/7.htm>

TABLE OF FIGURES

Fig. 1 Digital filter	3
Fig. 2 Example of decimation	5
Fig. 3 Interpolation	6
Fig. 4 FIR filter (order 3) implemented in direct-form	7
Fig. 5 FIR filter (order 3) implemented in symmetric direct-form	8
Fig. 6 FIR filter (order 3) implemented in transposed direct-form	8
Fig. 7 IIR filter (order 3) implemented in direct-form	9
Fig. 8 Software tool for choosing appropriate μ C	10
Fig. 9 Overview of Microchip μ C families	10
Fig. 10 Summary of PIC families performances	11
Fig. 11 Calculation speed of microcontrollers	12
Fig. 12 Code sequence used for Fig. 11	12
Fig. 13 Program Memory Window	13
Fig. 14 Disassembly Listing Window	13
Fig. 15 Simulator Trace Window	13
Fig. 16 DSP device performing the filter algorithm	15
Fig. 17 Maclaurin power series expansion for transcendental functions	19

TABLE OF EQUATIONS

Eq. 1 Non-recursive filters	3
Eq. 2 Recursive filters	3
Eq. 3 Discrete Fourier Transform	3
Eq. 4 Fast Fourier Transform	4
Eq. 5 Cosine Transform	4
Eq. 6 FIR filter (order 3) implemented in direct-form	7
Eq. 7 FIR filter (order 3) implemented in symmetric direct-form	8
Eq. 8 FIR filter (order 3) implemented in transposed direct-form	8